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Depinning from the defect located at a finite distance from the surface in the planar Abraham model

D N Mihajlović

Department of Physics and Meteorology, Faculty of Natural and Mathematical Sciences, 11001 Beograd, PO Box 550, Yugoslavia

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Abstract. A recently developed renormalisation group approach to interface pinning problems is generalised to deal with the depinning from the defect at a finite distance z removed from the surface in the planar Abraham model. The phase diagrams and the incremental defect free energies over the whole range of temperatures and distances z are obtained. The agreement with exact results, when available, is quite satisfactory.

1. Introduction

The binding of an interface by a line of defect bonds in the planar Abraham model is important to depinning (wetting) transitions, and has been extensively studied (Abraham 1980, 1981a, b, Burkhardt 1981, Chalker 1981, Kroll 1981, Chui and Weeks 1981, van Leeuwen and Hilhorst 1981, Švrakić 1983, Mihajlović and Švrakić 1983, Abraham and Švrakić 1986). In particular, an exact solution by Abraham (1980, 1981b) indicates that the defect adjacent to one surface of the lattice will bind an interface at low enough temperatures, but as the temperature is increased above a certain value T_W (below the bulk critical temperature T_c) the interface depins into the bulk. Exactly at T_W the depinning (wetting) transition takes place and the defect specific heat exhibits a jump discontinuity. If the defect is located in the interior of the lattice, however, depinning will not occur. In this case at the bulk critical temperature defect thermodynamics shows non-universal behaviour (Bariev 1979, McCoy and Perk 1980). We shall distinguish these two cases as: (i) the case with boundary defect and (ii) the case with the internal defect.

The purpose of this work is to generalise a position-space renormalisation group (RG) method applied earlier (Švrakić 1983, Mihajlović and Švrakić 1983) to the original wetting problem of Abraham (1980) to deal with the binding (and unbinding) of the interface by the defect at a finite distance removed from the surface of the planar Ising lattice. Although this problem represents a slightly generalised version of the original Abraham model, which interpolates between the cases (i) and (ii) above, a generalisation is needed in order to deal with the defects at the intermediate distances from the surface. This model we describe now.

Consider a square, two-dimensional, nearest-neighbour Ising model with periodic boundary conditions in one direction and antiperiodic in the other. Suppose, in addition, using z to label the position of the defect, with $z = 1$ describing the position adjacent to the surface, that a ladder of defect bonds is located at a distance z from the surface in the system, as shown in figure 1. Defect couplings K_d are parametrised

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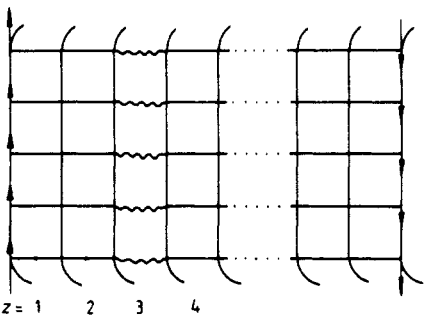

Figure 1. The geometry of the model considered. Couplings K_d and K are shown by wavy and full lines respectively. The numbers $z = 1, 2, \ldots$ label the position of the defect in order of increasing distance from the surface.

as $K_d = aK$ $(0 \leqslant a \leqslant 1)$, where the parameter a measures the amount by which defect couplings are weaker than bulk couplings K. Clearly, if (i) $z = 1$ this model corresponds to the original model for depinning from the boundary defect and (ii) when $z = \infty$ the defect is internal.

The remainder of this paper is organised as follows. In § 2 we motivate and develop our RG scheme. Section 3 presents a discussion of the wetting phase diagram. The results for the interface thermodynamic functions are presented in § 4. Section 5 contains our principal conclusions.

2. Position-space RG for wetting in the $d = 2$ Abraham model

The first and crucial question which we shall now discuss is the choice of our RG scheme which is dictated by the following observation. In order to understand a wide variety of phenomena observed in coexisting phases within the RG context, Klein *et al* (1976) have shown that any satisfactory RG scheme should have a low-temperature recursion formula of the form $K^{(n+1)} = b^{d-1}K^{(n)}$, up to exponentially small corrections, where b is the change in length scale of the RG transformation and d is the dimension of space. Since the wetting transitions are examples of coexistence singularities located along the bulk phase boundary, any appropriate RG scheme should display above low-temperature behaviour in order to adequately reproduce the physical content of the Abraham model. According to the above criteria, in what follows we shall consider only a first-order cumulant expansion approximation (see, e.g., Burkhardt and van Leeuwen 1982), where the new nearest-neighbour interaction is $K^{(n)} \times$ (number of bonds connecting the two cell blocks) for $K^{(n)}$ large, i.e., $K^{(n+1)} = b^{d-1}K^{(n)}$ as required. If the low-temperature recursion relation is $K^{(n+1)} = b^{d-1}K^{(n)} +$ some constant, as is the case in Migdal-Kadanoff (MK) and the cell-cluster approach, then this constant will accumulate with iterations and bring about a large error in the calculations of all thermodynamic properties of the Abraham model. Unfortunately, this is brought about by the majority rule projection operator which is used as standard in the cell-cluster approach, while in the MK scheme this constant takes into account the disconnected spins that result from the bond shifting. In order to improve the cell-cluster approach one would have to modify the projection operator and to make a careful choice of boundary conditions for the finite cluster as emphasised for the surface problems by Švrakić *et al* (1980).

We now consider how a position-space RG method has to be generalised in order to study the model just described. The generalised method is based on the cumulant expansion and proceeds in two stages: (i) the defect at a distance z is renormalised towards the surface in $n(z)$ steps as indicated in figure 2; (ii) the model we consider becomes the original Abraham model with the renormalised couplings, and therefore from that point on recursion relations obtained earlier (Švrakić 1983) are used with those couplings as the initial ones. To implement the calculation described above, the RG scheme, in the first stage, compels us to introduce new couplings K_1. These couplings belong to the horizontal bonds adjacent to the surface, with $z=1$. Let us then consider the defect at a distance z from the surface and let $z \in (2^{m-1}, 2^m]$, where $m \geqslant 1$. Now, the defect becomes a boundary one in $n(z)=m$ steps or $z(m)=1$, where $z(n)$ indicates generally the position of the given bond in the renormalised system after n iterations. The first-order cumulant RG recursion relations in the three-dimensional parameter space (K, K_d, K_1) are

$$K^{(n+1)} = 2K^{(n)}\langle S^{(n)}\rangle^2 \qquad 0 \leqslant n \leqslant m-1 \tag{1}$$

$$K_d^{(n+1)} = 2K_d^{(n)}\langle S^{(n)}\rangle^2 \qquad 0 \leqslant n \leqslant m-1 \tag{2}$$

$$K_1^{(n+1)} = 2K_1^{(n)}\langle S^{(n)}\rangle \qquad z(n)=1 \qquad 0 \leqslant n \leqslant m-2 \tag{3}$$

$$K_1^{(n+1)} = 2K_1^{(n)}\langle S_W^{(n)}\rangle\langle S^{(n)}\rangle \qquad z(n)=2 \qquad 0 \leqslant n \leqslant m-2 \tag{4}$$

$$K_d^{(m)} = 2K_d^{(m-1)}\langle S_W^{(m-1)}\rangle\langle S^{(m-1)}\rangle \tag{5}$$

where $\langle S^{(n)}\rangle$ is the nth iterate of the average spin in the basic 2×2 block in the bulk and $\langle S_W^{(n)}\rangle$ is the nth iterate of the average spin in the block immediately adjacent to

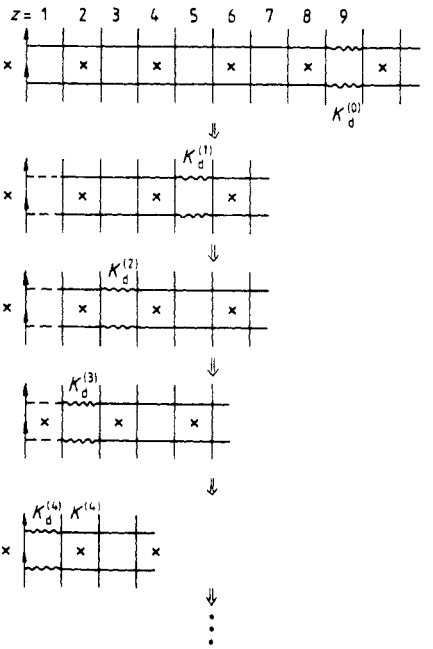

Figure 2. Successive steps of the generalised RG procedure for the defect at the distance $z=9$ $(m=4)$ as explained in the text. Couplings K, K_d and K_1 are shown by full, wavy and broken lines respectively.

the surface which is topologically equivalent to the basic 3×1 block on the triangular lattice due to the boundary conditions. With the modified majority rule projection operator (Švrakić 1983) we have $\langle S^{(n)}\rangle=(\exp(4K^{(n)})+p)/(\exp(4K^{(n)})+6+\exp(-4K^{(n)}))$ where p is some adjustable parameter and

$$\langle S_{\mathrm{W}}^{(n)}\rangle=\exp(2K_1^{(n)}+K^{(n)})/(\exp(2K_1^{(n)}+K^{(n)})+2\exp(-K^{(n)})+\exp(-2K_1^{(n)}+K^{(n)})).$$

With $p=p_{\mathrm{e}}=4\sqrt{2}-3$, the bulk recursion relation (1) yields the exact critical coupling $K_{\mathrm{c}}=\frac{1}{2}\ln(1+\sqrt{2})$. The latter choice of parameter is convenient as long as one is interested in the global behaviour. The second stage of the RG scheme starts in a point $(K^{(m)}, K_{\mathrm{d}}^{(m)})$ in the (K, K_{d}) plane after the first m iterations. The recursion relations for this stage of the renormalisation procedure are (Švrakić 1983)

$$K^{(m+n)}=2K^{(m+n-1)}\langle S^{(m+n-1)}\rangle^2 \tag{6}$$

$$K_{\mathrm{d}}^{(m+n)}=2K_{\mathrm{d}}^{(m+n-1)}\langle S^{(m+n-1)}\rangle \tag{7}$$

where $n\geqslant1$. In figure 2, it is indicated how the renormalisation scheme proceeds in the both stages for the $z=9$ $(m=4)$.

3. Wetting phase diagrams

The wetting transition phase diagram is obtained from the global behaviour of the RG flows in the parameter space (K_{d}, K). The way in which our recursion relations lead to this phase diagram can be obtained from simple arguments based on the following important property of equations (6) and (7). A closer inspection of these equations shows the existence of a transient region in which, because of the inequality $\langle S^{(m+n)}\rangle^2<\langle S^{(m+n)}\rangle<1$, $K_{\mathrm{d}}^{(m+n)}$ will grow with iterations faster than $K^{(m+n)}$. It can be seen that, even though initially $K_{\mathrm{d}}^{(m)}<K^{(m)}$, the flows will be such that after a certain number of transient iterations n the coupling $K_{\mathrm{d}}^{(m+n)}$ may exceed the coupling $K^{(m+n)}$. However, if the value of the coupling $K^{(m)}$ is sufficiently large then $\langle S^{(m+n)}\rangle$ will approach unity after only a few iterations and $K_{\mathrm{d}}^{(m+n)}$ will not exceed $K^{(m+n)}$ since they grow by a factor of two at each step near the fixed point at $T=0$. Therefore, after infinitely many iterations one either obtains $K_{\mathrm{d}}^{(\infty)}<K^{(\infty)}$, indicating that the interface is pinned to the defect or $K_{\mathrm{d}}^{(\infty)}>K^{(\infty)}$ when the interface is depinned.

The wetting transition occurs when (Švrakić 1983)

$$\lim_{n\to\infty}(K^{(m+n)}-K_{\mathrm{d}}^{(m+n)})=0 \tag{8}$$

giving the wetting transition temperature $T_{\mathrm{W}}(a, z)$, for a given value of defect parameter a and distance z, as the initial value of K. The RG phase diagram, for the defect at a distance z from the surface, is calculated by iterating equations (6) and (7):

$$K^{(m+n)}=2^{m+n}K\prod_{i=0}^{m+n-1}\langle S^{(i)}\rangle^2 \tag{9}$$

$$K_{\mathrm{d}}^{(m+n)}=2^{m+n}aK\prod_{i=0}^{m-2}\langle S^{(i)}\rangle^2\langle S^{(m-1)}\rangle\langle S_{\mathrm{W}}^{(m-1)}\rangle\prod_{i=m}^{m+n-1}\langle S^{(i)}\rangle \tag{10}$$

thus giving (by equating equations (9) and (10))

$$a(z)=\lim_{N\to\infty}\prod_{n=0}^{N}\langle S^{(n)}\rangle\left[\left(\prod_{n=0}^{m-2}\langle S^{(n)}\rangle\right)\langle S_{\mathrm{W}}^{(m-1)}\rangle\right]^{-1} \tag{11}$$

which is convergent since $\langle S^{(n)}\rangle \to 1$ as $n \to \infty$ for any $T < T_c$. Figure 3 shows the depinning transition phase diagram obtained from (11). Note that the phase diagram exhibits quite resonable physical behaviour: as z increases, the region above the curves, where the interface is depinned, reduces and vanishes as $z \to \infty$. This means that as z increases the defect becomes more and more efficient at pinning the interface and that for $z = \infty$, the case of the internal defect, the interface will be pinned for all temperatures and depinning will not occur, in agreement with the exact result of Abraham (1981b). In order to show that equation (11) encompasses cases (i) and (ii) mentioned above, it is useful to describe the picture behind it. It is easy to see that the contributions to the numerator in (11) come from both stages of the RG scheme and that all members of the sequence $\{\langle S^{(n)}\rangle\}$ are associated with the blocks to the right of the defect. The contributions to the denominator in (11) can only come from the first stage and the first $(m-1)$ members of the sequence $\{\langle S^{(n)}\rangle\}$ and $\langle S_W^{(m)}\rangle$ are associated with the blocks to the left of the defect. For $z = 1$, we have $\langle S\rangle = 1$ for the block immediately to the left of the defect, due to the boundary conditions, and (11) reduces to

$$a(1) = \lim_{N\to\infty} \prod_{n=0}^{N} \langle S^{(n)}\rangle \tag{12}$$

which is just equation (9) of Švrakić (1983). Now, in the case of the internal defect $a(\infty) \to 1$, since $m \to \infty$ all members of the sequence$\{\langle S^{(n)}\rangle\}$ contribute to the denominator in (11), with the implication that the interface always remains pinned. We can also quickly deal with the case of generalised wetting (Švrakić 1983), if we let all the bonds to the right of the defect to have values K_2 and those to the left, values K_1 with the obvious substitutions $\langle S\rangle \to \langle S_2\rangle$ and $\langle S\rangle \to \langle S_1\rangle$ respectively. With $K_1 \neq K_2$, we shall take $z = \infty$ and since all members of the sequence $\{\langle S_2^{(n)}\rangle\}$ and $\{\langle S_1^{(n)}\rangle\}$ contribute to the numerator and denominator in (11) respectively, (11) reduces to

$$a(\infty) = \lim_{N\to\infty} \prod_{n=0}^{N} (\langle S_2^{(n)}\rangle / \langle S_1^{(n)}\rangle) \tag{13}$$

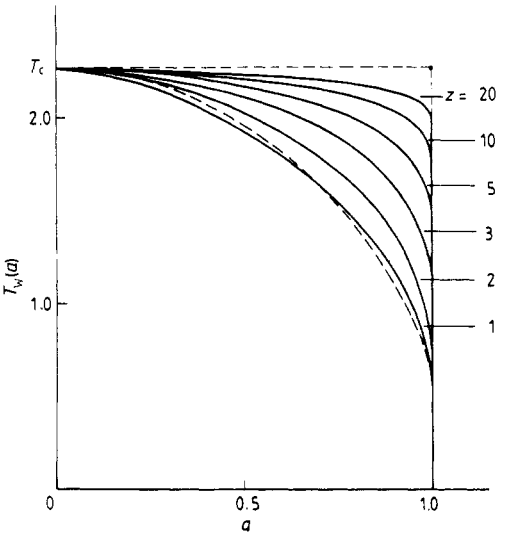

Figure 3. Depinning transition phase diagram obtained from RG calculation, equation (9). The broken curve is the exact result (Abraham 1980) for $z = 1$. The interface is pinned in the regions under the curves.

which is just equation (14) of Švrakić (1983), and in a good agreement with the exact result (Abraham and Švrakić 1986) indicating that the interface unbinds from the internal defect whenever $K_1 \neq K_2$.

4. Defect thermodynamic functions

We now turn to the calculation of defect thermodynamic functions. In particular the incremental defect free energy $f_d(K, K_d, z)$ is obtained from the ratio (Abraham 1980)

$$f_d(K, K_d, z) = \lim_{N, N_d \to \infty} \frac{1}{N_d} \ln \frac{Z_{+-}}{Z_{++}} \tag{14}$$

where Z_{+-} is the partition function for our model with antiperiodic boundary conditions, while Z_{++} is that for the model with periodic boundary conditions. Using definition (14) and recursion relations (6) and (7) we get (Mihajlović and Švrakić 1983)

$$f_d(K, K_d, z) = \lim_{n \to \infty} \frac{\max(-2K^{(m+n)}, -2K_d^{(m+n)})}{2^{m+n}}. \tag{15}$$

Below the depinning transition temperature $T_w(a, z)$, $K_d^{(m+n)} < K^{(m+n)}$ for all values of n. The defect free energy takes the usual Onsager (1944) values for temperatures $T > T_w(a, z)$, in agreement with the exact result of Abraham (1980). In figure 4 we show $f_d(K, K_d, z)$ divided by $-2K$, obtained from equation (15). Note that $f_d(K, K_d, z)$ merges with Onsager's surface tension at $T_w(a, z)$ and that $f_d(K, K_d, z)$ approaches T_c always with the same angle independent of $a(z)$ for finite z showing universal behaviour and that $f_d(K, K_d, z)$ approaches T_c with an angle depending on $a(\infty)$ (Mihajlović and Švrakić 1983), i.e. behaviour at $T = T_c$ is non-universal (McCoy and Perk 1980).

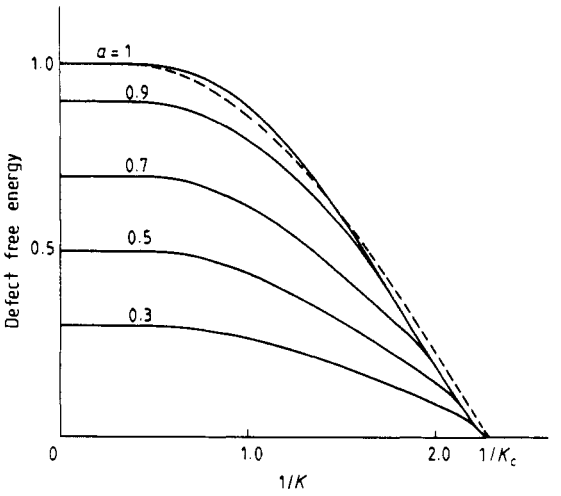

Figure 4. Incremental defect free energy for the defect at the position $z = 3$, obtained via the RG method from (13). The broken curve is the exact result for Onsager's surface tension.

5. Conclusions

We have generalised a position-space RG scheme introduced previously by Švrakić (1983) to the investigation of (i) wetting phase diagrams and (ii) interface thermodynamics of the planar Abraham model with the defect removed from the surface. The phase diagrams and the incremental defect free energies over the whole range of temperatures and distances z are obtained. The agreement with exact results (when available) is quite satisfactory. It is to be noted that all quantities we have calculated are smoothly dependent on z and behave just as one qualitatively expects. We conclude that the position-space RG method presented in this work can be successfully applied to interface problems. Further extension of this method to more interesting $d = 3$ systems appears feasible.

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